Outcome 2
LESSON 9
Initial Transverse Metacentre

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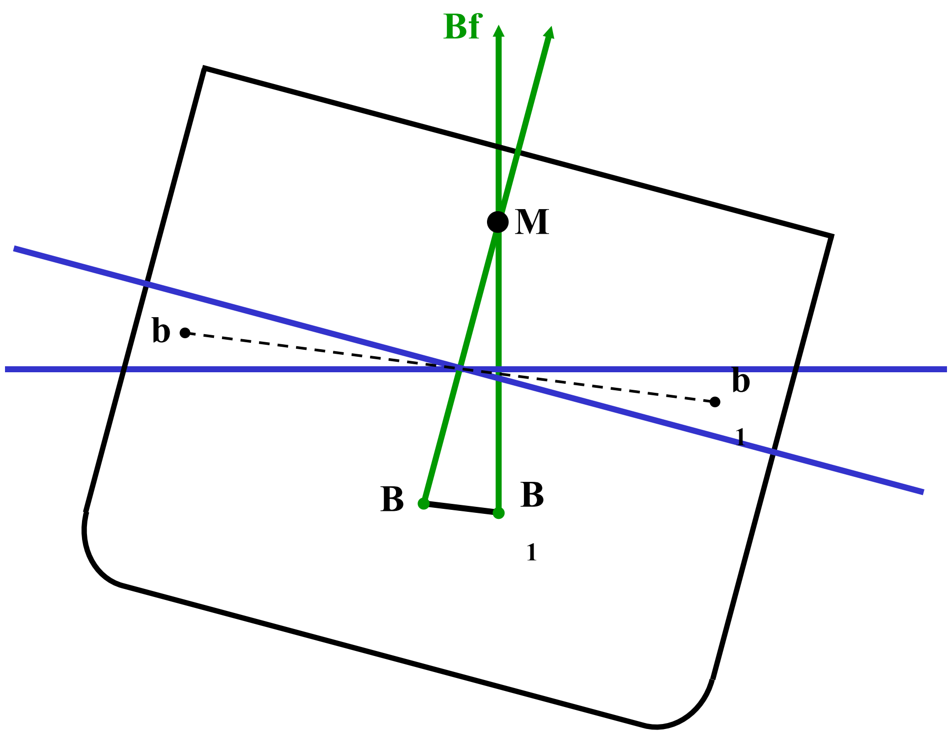
INITIAL TRANSVERSE METACENTRE

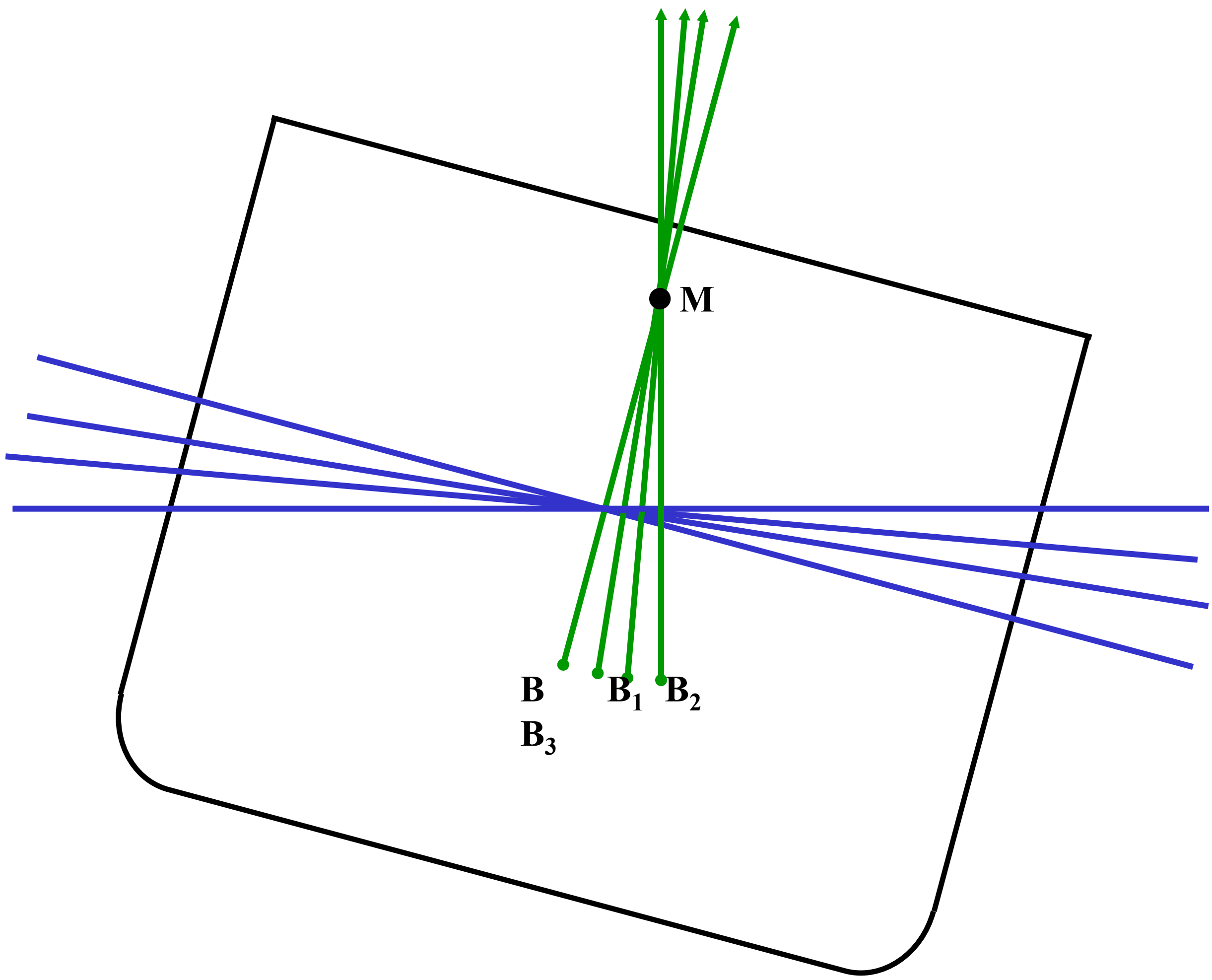
This is the point of intersection of the lines of action of buoyancy force (Bf) when the ship is in the upright and subsequently heeled conditions ***within small angles of heel.***



$$BB_1 = \frac{v \times d}{V}$$

where **v** is the volume of the transferred wedge of buoyancy and **V** is the ship's volume of displacement.

If B is plotted for several small angles of heel it may be assumed that it follows the arc of a circle centred at M.



BM is termed the ***metacentric radius*** and the height of the initial transverse metacentre (KM) may be calculated thus:

$$\mathbf{KM = KB + BM}$$

Calculating KM for box-shaped vessels

For a box-shaped vessel on an even keel:

$$\boxed{KB = \frac{\text{draught}}{2}}$$

and;

$$\boxed{BM = \frac{I}{V}}$$

where **I** is the moment of inertia (second moment of area) of the WPA and **V** is the volume of displacement of the box-shaped vessel.

For a box-shaped vessel: $I = \dfrac{LB^3}{12}$

where **L** and **B** are the length and breadth of the WPA respectively.

Therefore:

$$\boxed{BM = \frac{LB^3}{12V}}$$

Thus:

$$\boxed{KM_{BOX} = \frac{\text{draught}}{2} + \frac{LB^3}{12LBd}}$$

Note that this formula can obviously be simplified further!

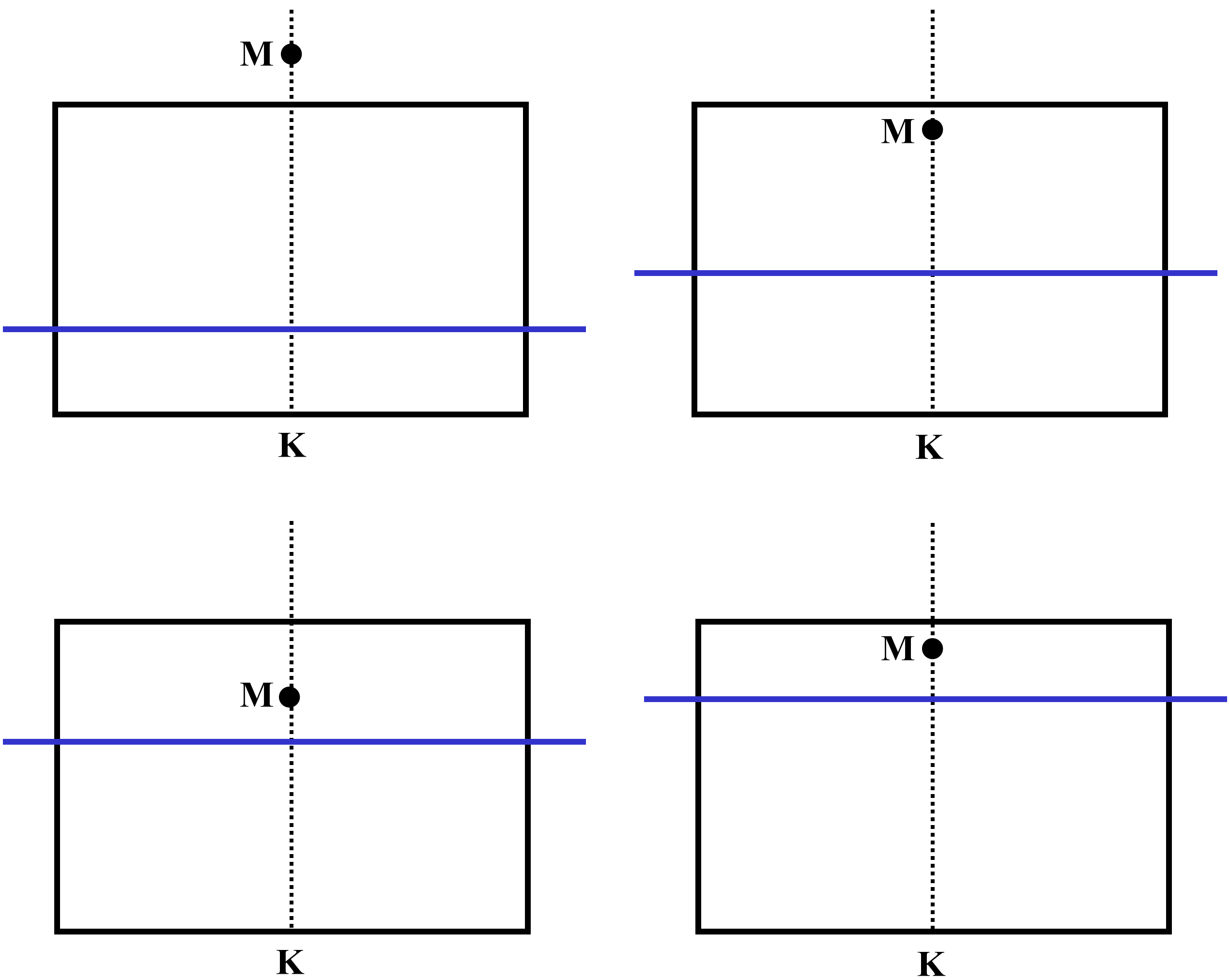
SAQ

Prove that the KM of a box-shaped vessel changes with draught as shown below for the range of draughts 1.00 m to 15.00 m given that:

length = 100 m
breadth = 20 m.

Tip

Use the formulae to find values of KB and BM and then sum these to find the corresponding KM values.



Answer

The values for KM are shown below having been calculated using: $KM = KB + BM$ ***where:***

$$KB = \frac{draught}{2}; \qquad \text{and} \qquad BM = \frac{B^2}{12d}$$

Draught (m)	KB (m)	BM (m)	KM (m)
1	0.5	33.33	33.83
2	1	16.67	17.67
3	1.5	11.11	12.61
4	2	8.33	10.33
5	2.5	6.67	9.17
6	3	5.56	8.56
7	3.5	4.76	8.26
8	4	4.17	8.17
9	4.5	3.70	8.20
10	5	3.33	8.33
11	5.5	3.03	8.53
12	6	2.78	8.78
13	6.5	2.56	9.06
14	7	2.38	9.38
15	7.5	2.22	9.72

KM REDUCING (draughts 1 to 8)

KM INCREASING (draughts 8 to 15)

SAQ

Plot the values of KM with the X-axis labelled 'draught' and the Y-axis labelled 'KM' and 'KG'. Plot a value of KG = 9.00 m on the graph.

The graph is plotted at the end of these notes.

SAQ

Use the graph you have drawn to determine the following:

(a) ***The minimum value of KM and the draught at which it occurs;***

(b) ***The range of draughts at which the vessel will be unstable;***

(c) ***The righting moment when the box is heeled to an angle of 5° when the upright draught is 3.00 m in salt water (RD 1.025).***

Answer

(a) *The minimum value of KM = 8.00 m and occurs at draught = 8.00 m.*

(b) *The range of draughts at which the vessel will be unstable is between 5.20 m and 12.50 m.*

(c) *At a draught of 3.00 m KM was calculated to be 12.61 m.*

$$\begin{array}{ll} KM & 12.61\ m \\ \underline{KG} & \underline{9.00\ m} \\ \underline{GM} & \underline{3.61\ m} \end{array}$$

$$\text{Displacement} = L \times B \times d \times \text{density}$$

$$\text{Displacement} = 100 \times 20 \times 3.00 \times 1.025$$

$$\text{Displacement} = 6150 \text{ tonnes}$$

$$GZ = GM \times \sin\theta$$

$$GZ = 3.61 \times \sin 5°$$

$$GZ = 0.31463....$$

Therefore:

$$\text{Righting moment} = GZ \times \text{Displacement}$$

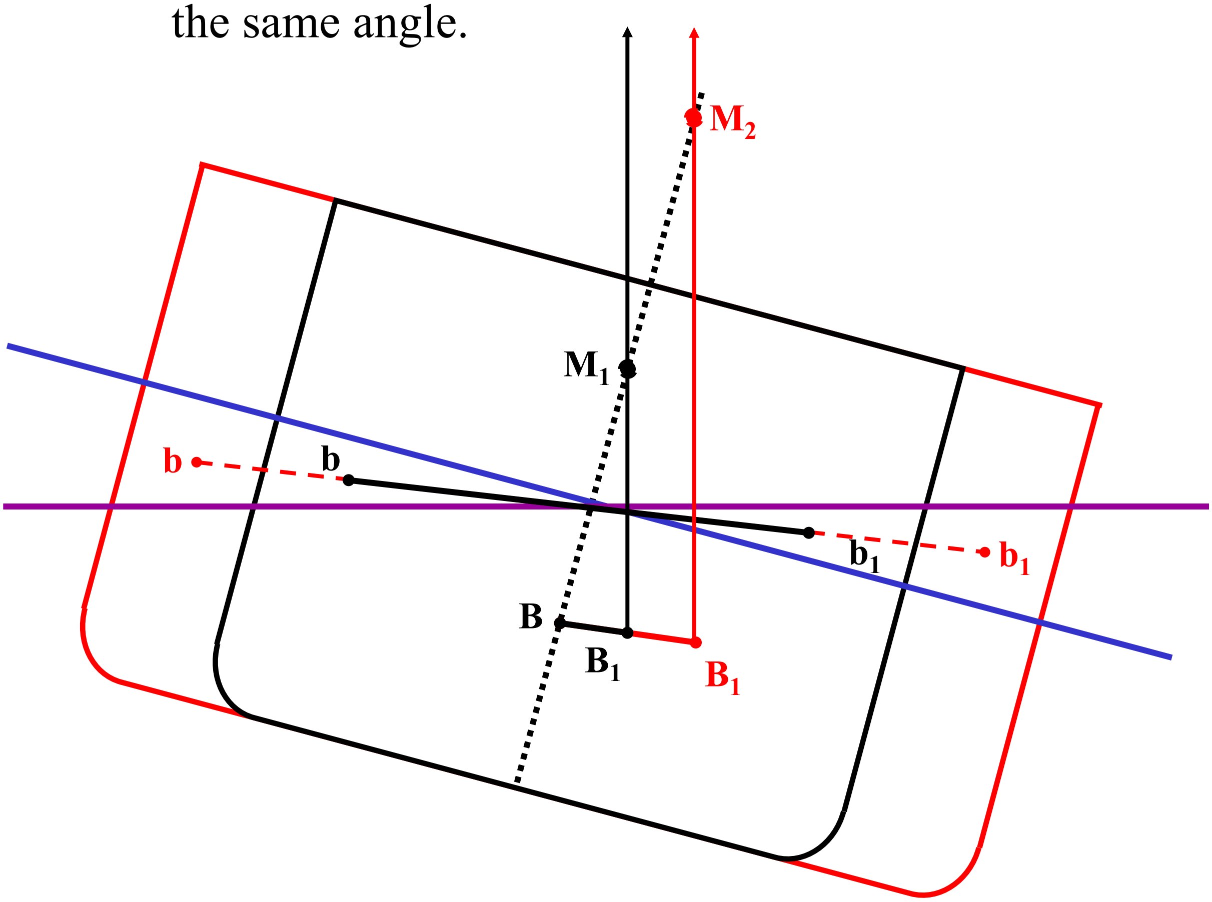
$$\text{Righting Moment} = 0.31463.. \times 6150$$

$$\text{Righting moment} = \underline{1935 \text{ t-m}}$$

FACTORS AFFECTING KM

1. **Beam**

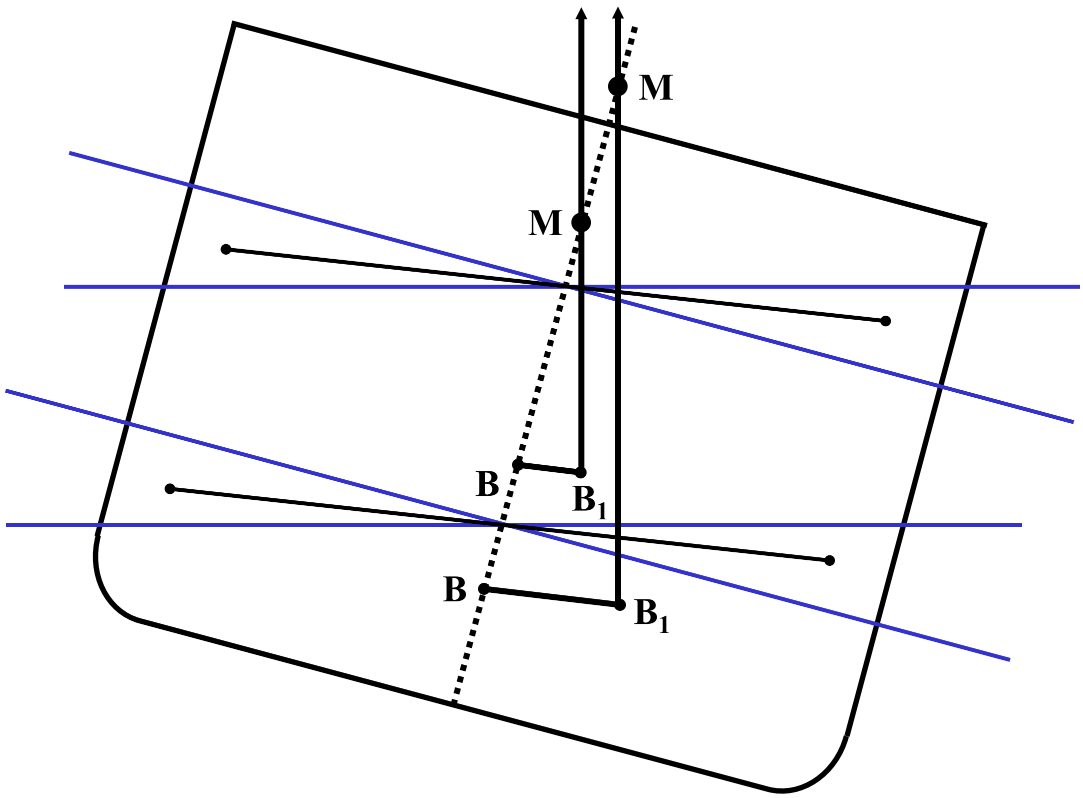
Consider two ships of different beam each heeled to the same angle.



In the narrow ship a wedge of buoyancy is transferred from the high side to the low side (**bb$_1$**) causing **B** to move to **B$_1$**. Metacentre is at **M$_1$**.

In the broader ship a ***larger*** wedge of buoyancy is transferred from the high side to the low side (**bb$_1$**) causing **B** to move further out to **B$_1$**. Metacentre is higher at **M$_2$**.

2. Draught (Displacement)



Consider the formula: $$\boxed{BB_1 = \frac{v \times bb_1}{V}}$$

At the load draught the volume of the transferred wedge of buoyancy (**v**) represents a smaller part of the total volume of displacement of the ship (**V**) than at the light draught.

Thus, KM decreases as draught increases for the normal range of operational draughts of a ship.